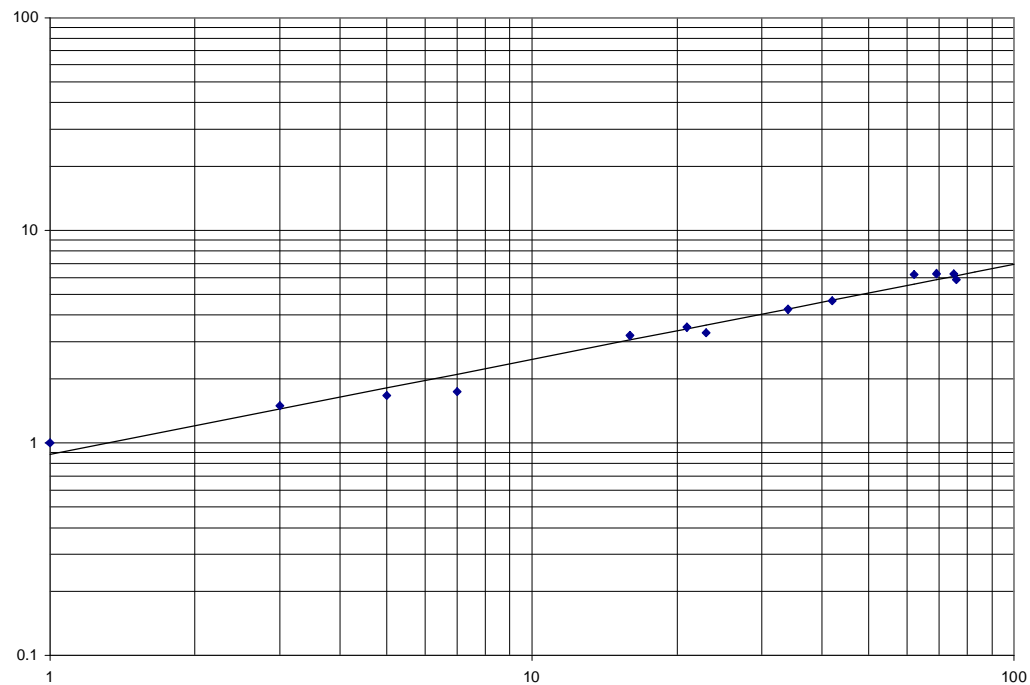


RELIABILITY GROWTH



(13)

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Reliability Growth

- The improvement in a reliability figure of merit (MTBF, MMBF, λ etc.) caused by successfully learning about and correcting of faults in a products design, use or manufacture. This is accomplished through planned testing, and problem identification and removal (TAAF), etc.

Benefits

- **Provides a metric to evaluate the improvement in design and manufacture of a product.**
- **Aids in evaluation the corrective action program on equipment.**
- **Provides an effective quantitative measurement of the effectiveness of the reliability program.**
- **Used as an oversight/program management tool.**

Objective

- **Be able to answer or perform:**
- **What is the basic approach methodology to reliability growth models?**
- **Apply and understand the Duane reliability growth model.**
- **Understand the AMSAA reliability growth model.**

Outline

- **Background**
- **Reliability Growth Mathematics**
- **Modeling Strategy**
- **Duane Reliability Growth Model**
- **Army Material Systems Analysis Activity (AMSAA) Reliability Growth Model**

Background

- New products (except prototypes) have often been less reliable during early development and early production.
- Tests and service brings to light problems which are analyzed, corrected and incorporated into subsequent production runs (or incorporated into all models as a result of warranty campaigns, recalls etc).
- Therefore products exhibit reliability growth.
- This was first “formally” analyzed by J.T.Duane.
- It was found that the log (cumulative failure rate) vs. log (cumulative time) is linear.

Reliability Growth Mathematical Relationships

$$\lambda_c = F_c / (t_{test}) = kt^{-\alpha} \text{ where:}$$

λ_c = cumulative failure rate = tot. failure/tot. hrs.

F_c = cumulative failures

t_{test} = tot. test hours

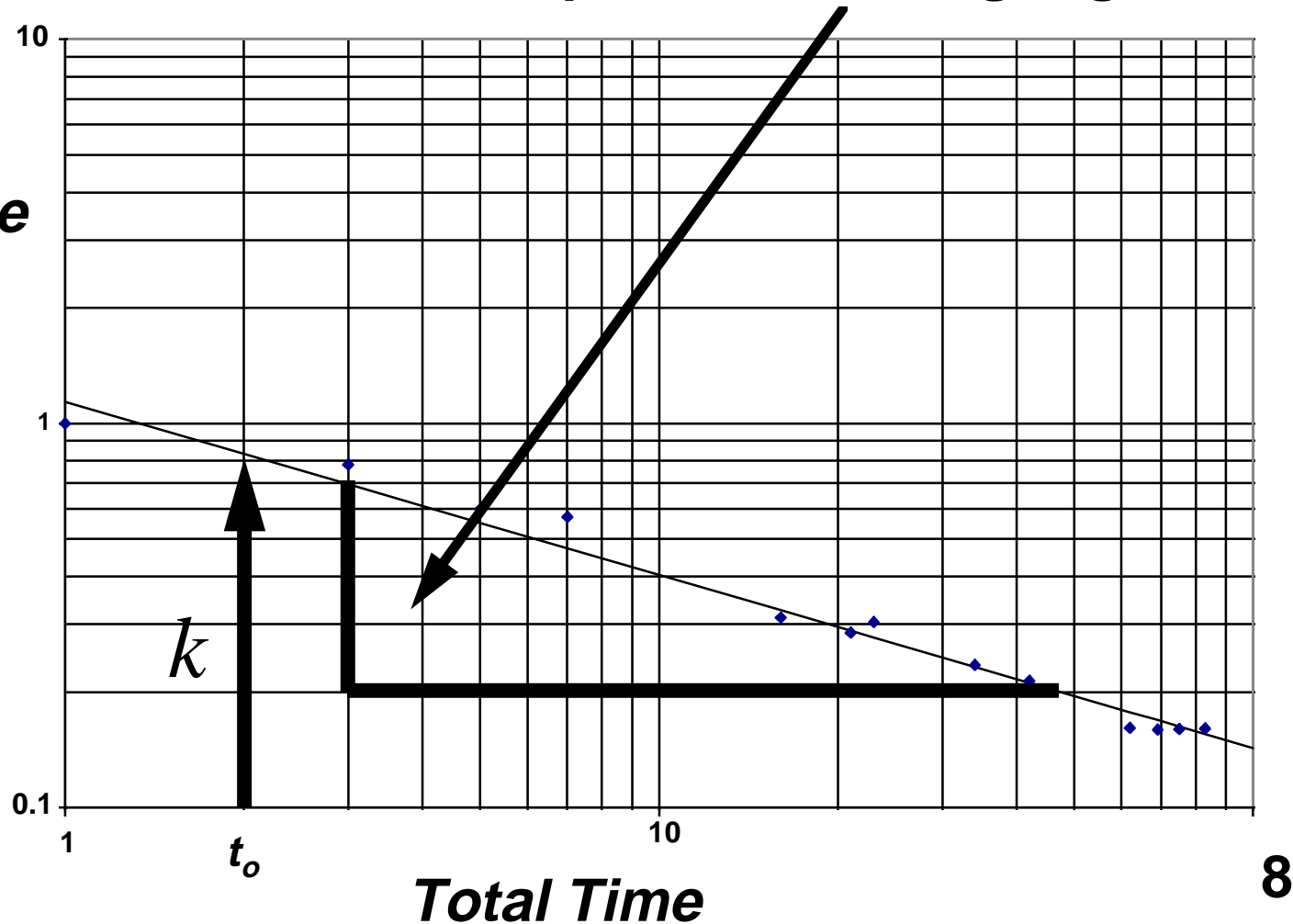
α = growth rate (slope of line on log-log scale);
a function of equipment complexity, design.

k = constant; a function of complexity, design life,
reliability standards, etc.

Reliability Growth Mathematical Relationships

α = slope of line on log-log scale

$\lambda_c =$
tot. failure
/tot. hrs.



Reliability Growth Mathematical Relationships

(continued)

Also instantaneous, λ_i , rate is given by:

$$\lambda_i = (1-\alpha) \lambda_c$$

similarly instantaneous MTBF, $MTBF_i$ is given by:

$MTBF_i = 1/\lambda_i$ and $MTBF_c = 1/\lambda_c$ therefore:

$$MTBF_i = MTBF_c / (1- \alpha)$$

where $MTBF_c$ = cumulative MTBF

Reliability Growth Mathematical Relationships

(continued)

$$\log (MTBF_c) = \log (MTBF_{c0}) + \alpha \log (t-t_0)$$

where

$MTBF_c$ = cumulative MTBF
= total hours / total failures

$MTBF_{c0}$ = initial cumulative MTBF

t = time

t_0 = initial time

α = growth rate or slope of line.

Duane Model

- **Learning curve approach**
 - Lessons learned should result in reliability growth
- **Continuous Model**
 - Used to monitor failure rate vs. time
- **Developed to monitor growth during TAAF phase**

Fundamental Assumption of Duane

- Reviewing formulas we have:

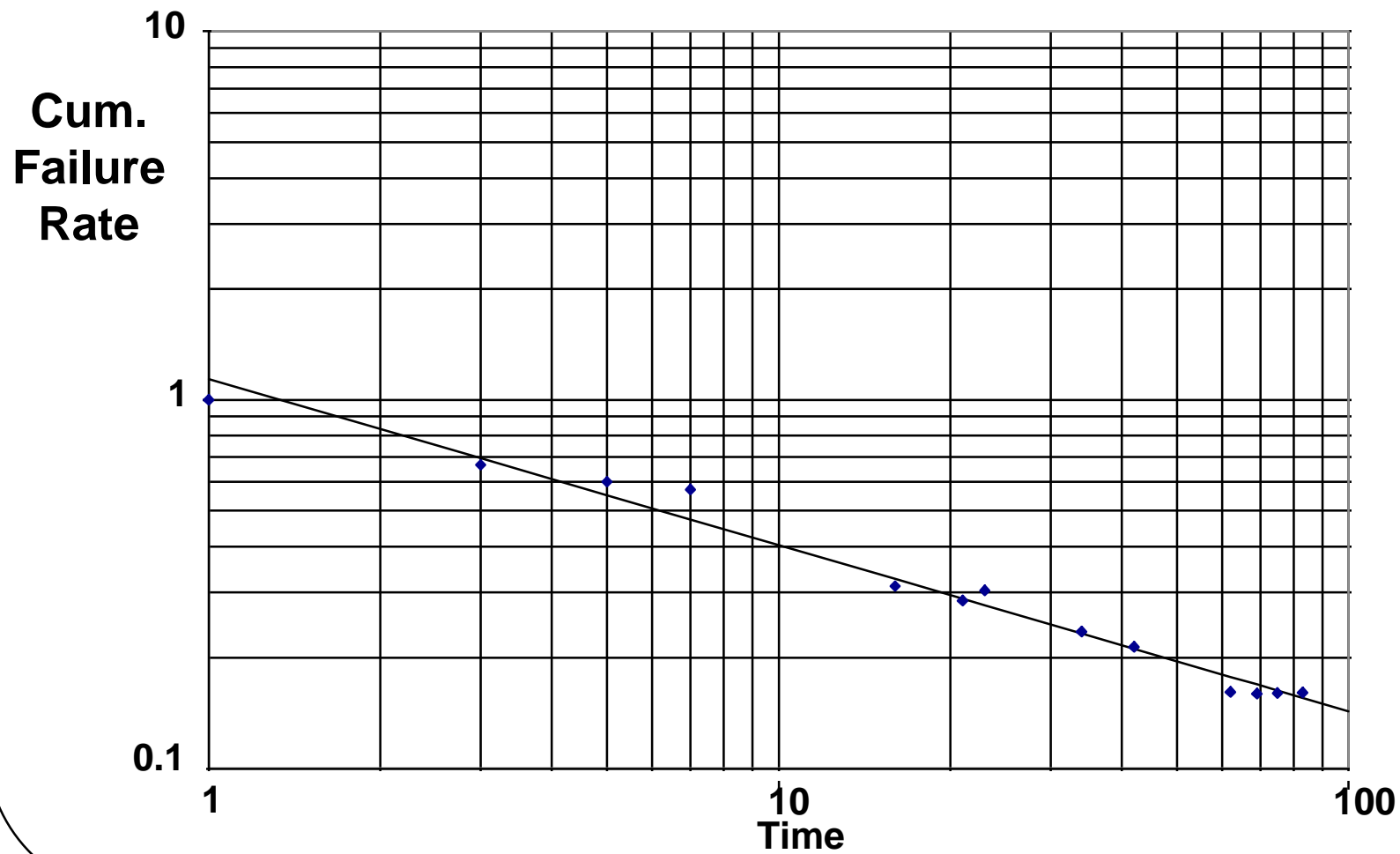
$$\lambda_c = F_c / (t_{test}) = kt^{-\alpha}$$

$$MTBF_c = 1/\lambda_c \text{ and } MTBF_i = 1/\lambda_i$$

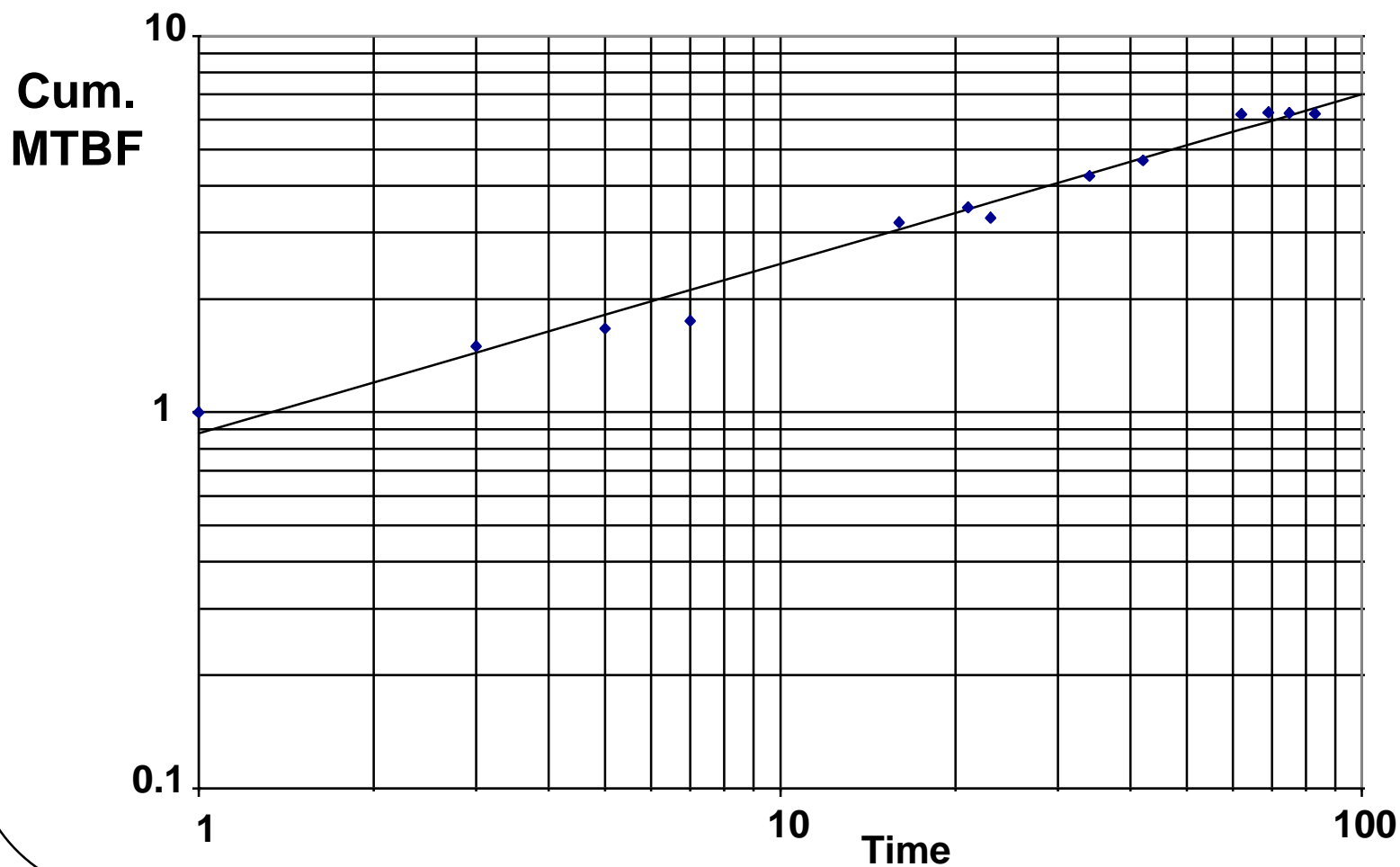
$$MTBF_i = MTBF_c / (1 - \alpha)$$

$$\log(\lambda_c) = \log(k) - \alpha \log(t)$$

Duane Model Plot



Positive Slope Duane Plot



Performing Reliability Growth Analysis -- Current Data

- Use MTBF (the line goes up instead of down)
- Plot the raw achieved MTBF for each test sample. It helps people grasp the concept if they see raw data.
- Plot Cumulative MTBF vs. total test time and draw a best fit line through the data.
- Estimate the reliability growth slope.
- As soon as a reasonable chart line can be found, plot the instantaneous MTBF
- Extend the chart line for a forecast.

AMSAA Model

Continuous model

Normally used to monitor failure rate vs. time

May be used on “one-shot” Items

Large sample required

Used to monitor reliability growth during TAAF

Method developed to see effects of design changes over short duration.

AMSAA Model

- The intensity function can be approximated by the Weibull failure rate function:

$$\rho(T) = \lambda \beta t^{\beta-1}$$

where: $\rho(T)$ = cumulative failure rate

$\lambda > 0$, scale parameter

$\beta > 0$, shape parameter

t = cumulative test time

AMSAA Model

- The maximum likelihood method for time truncated test provides a point estimate of as follows:

$$\beta = \frac{N}{N \ln(T) - \sum_i \ln(X_i)}$$

Where: N = number of failures

T = cumulative test time

X_i = time of failure

$i = 1 \dots n$

AMSAA Model

- For small sample sizes it is recommended that the unbiased estimator of B be used:

$$\beta_{unbiased} = \frac{N-1}{N} \beta$$

AMSAA Model

- The point estimate of the scale parameter is determined by:

$$\lambda = \frac{N}{T^\beta}$$

where: N = number of failures

T = cumulative test time

λ = scale parameter

β = point estimate or unbiased estimator

AMSAA Model

- The point estimate $\rho(T)$ is calculated by:

$$\rho(T) = \beta \frac{N}{T}$$

Where:

$\rho(T)$ = point estimate of intensity function

β = point estimate or unbiased estimator

N = the number of failures

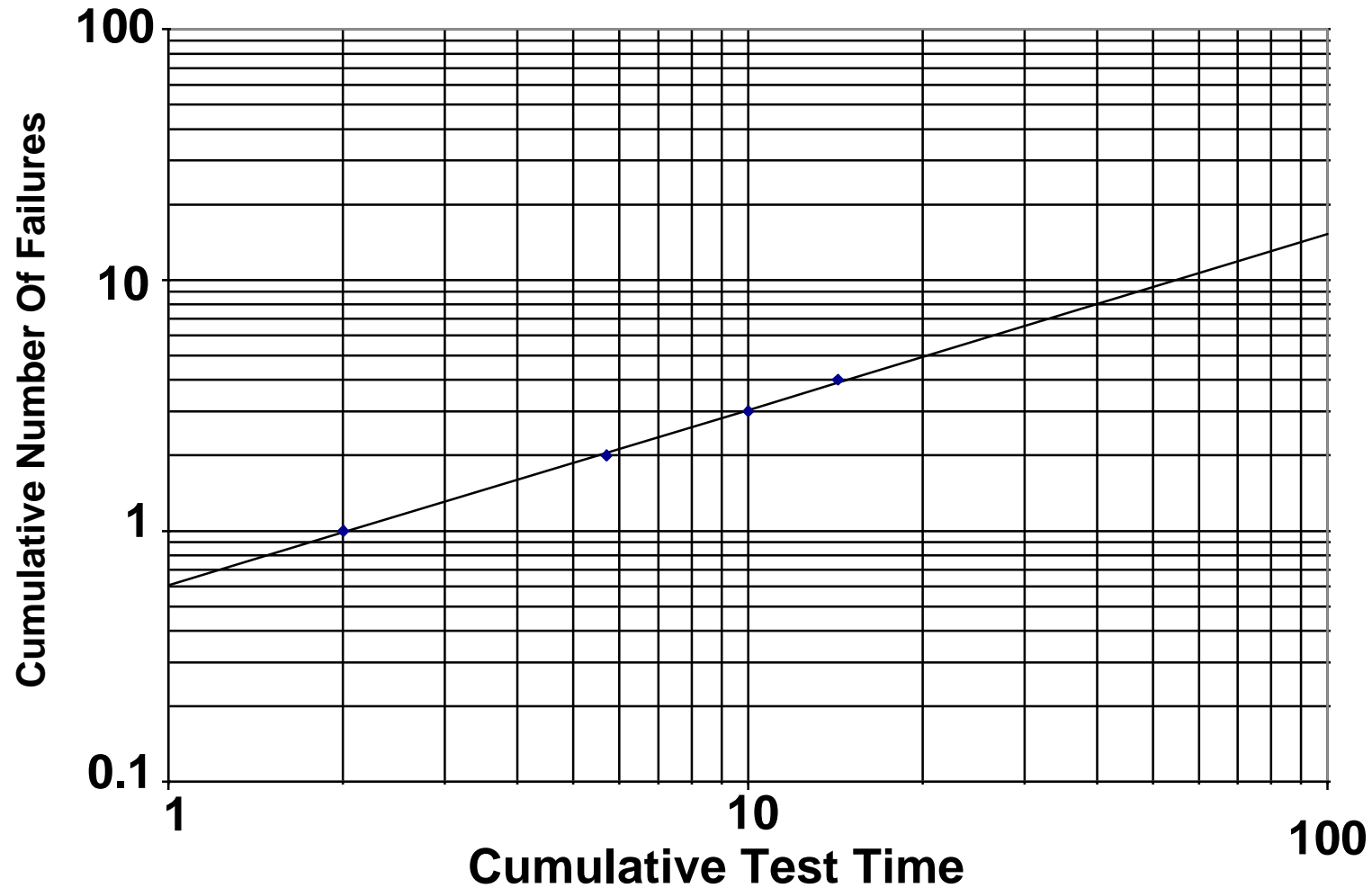
T = the total test time

AMSAA Model

- The point estimate of the Reliability is equal to $1 - \rho(T)$. A confidence interval can be found for reliability based on the number of failure points and interval desired as was found above for MTBF.

$$1 - \frac{1}{MTBF_{(L)}} \leq 1 - \rho(T) \leq 1 - \frac{1}{MTBF_{(U)}}$$

AMSAA Cumulative Failure Plot



Conclusions:

- **Reliability Growth Analysis is an effective means to measure the reliability and design improvement effort.**
- **Cumulative MTBF (or MMBF) is plotted against total test time on a log - log scale which plots as a straight line.**
- **The slope of the line can be an indication of the reliability effort.**
- **The Duane Model is a continuous reliability growth model used to monitor failure rate vs. time. It was developed to monitor growth especially during TAAF phase.**
- **The AMSAA model is also a continuous model often used to monitor failure rate vs. time. It is especially effective during TAAF and over short durations.**
- **The AMSAA model may be used on “one-shot” Items but a large sample required.**

Additional Information

F

Find the Growth Rate of an Experimental Sensor:

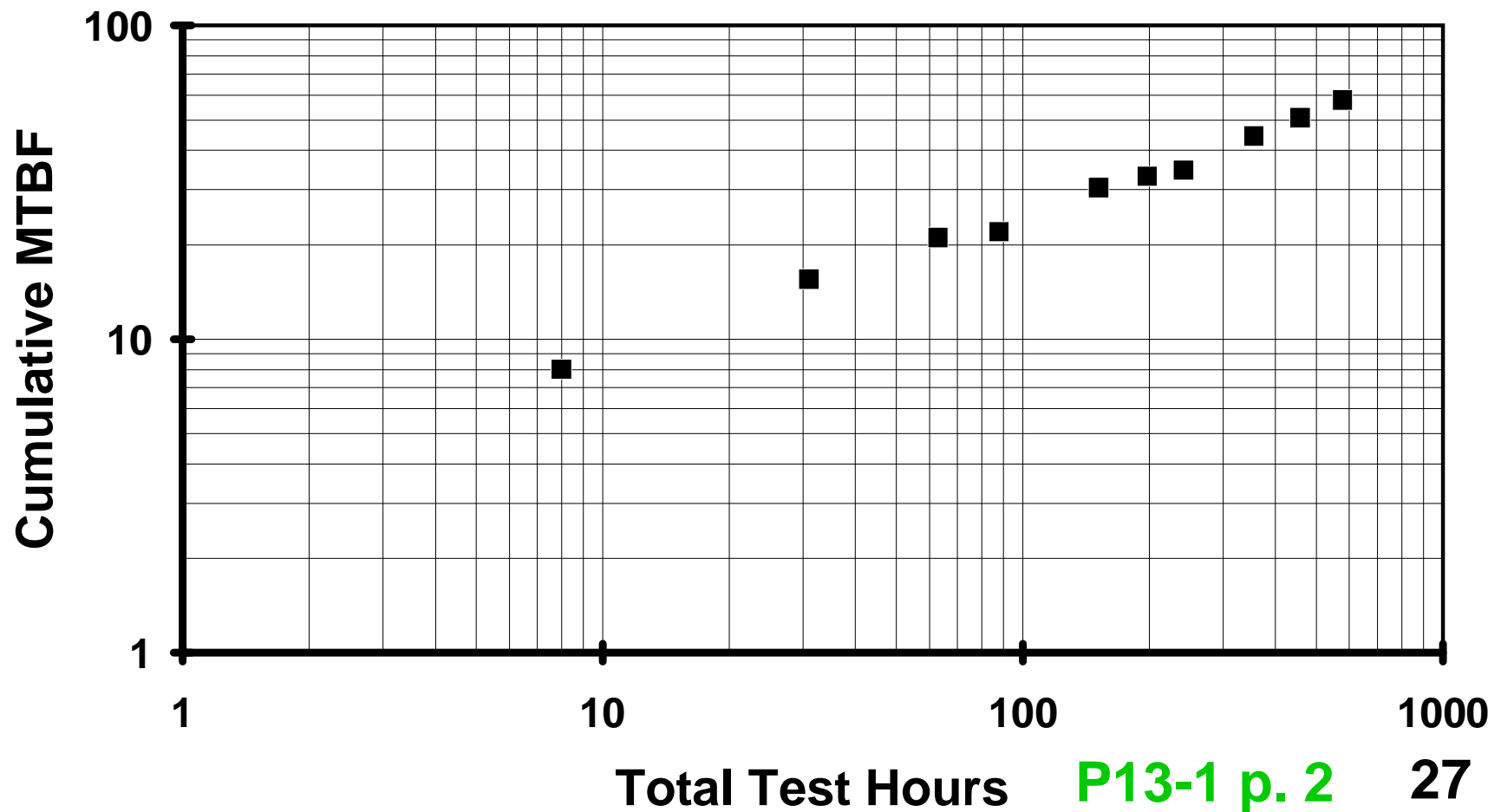
• Test Hours	Failure Number	Cum. λ	Cum MTBF
• 8	1	_____	_____
• 31	2	_____	_____
• 63	3	_____	_____
• 88	4	_____	_____
• 152	5	_____	_____
• 198	6	_____	_____
• 242	7	_____	_____
• 355	8	_____	_____
• 458	9	_____	_____
• 579	10	_____	_____

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Sensor Reliability Growth



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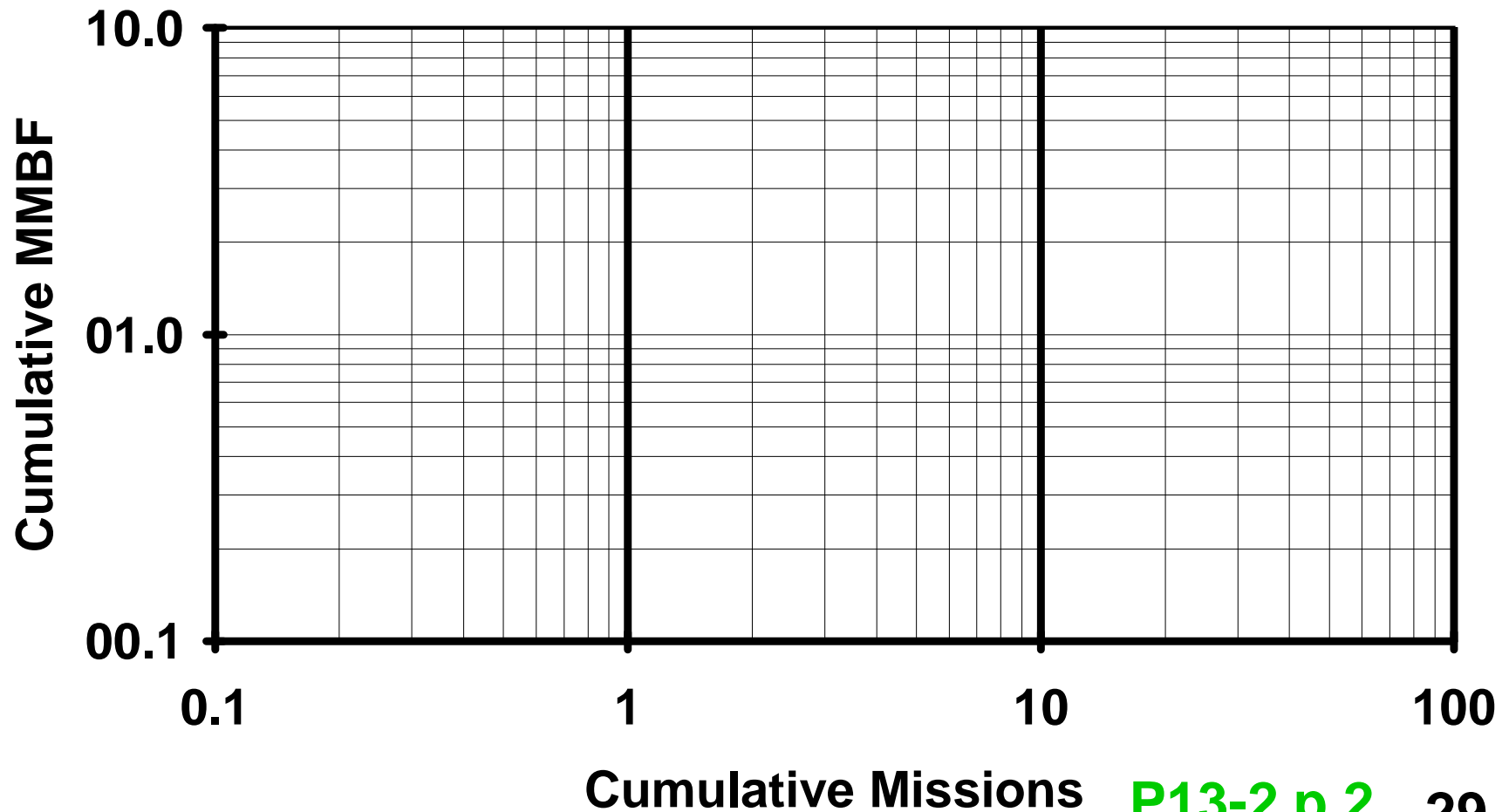
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Find the Reliability Growth of the Following Missile:

• Cum. • Missions	Fail No.	Cum. λ	Cum MTBF
• 1	1	_____	_____
• 3	2	_____	_____
• 4	3	_____	_____
• 7	4	_____	_____
• 16	5	_____	_____
• 21	6	_____	_____
• 23	7	_____	_____
• 34	8	_____	_____
• 42	9	_____	_____
• 62	10	_____	_____
• 69	11	_____	_____
• 75	12	_____	_____
• 76	13	_____	_____

F

Missile Reliability Growth



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Reliability Growth Mathematical Relationships

also: (continued)

$$\lambda_i = (1-\alpha) \lambda_c$$

where:

λ_i = instantaneous failure rate

This can be shown from:

$$\lambda_i = \lambda(t) = \lim_{\Delta t_{test} \rightarrow 0} (\Delta F_c / \Delta t_{test}) = \delta F_c / \delta t_{test} = (1-\alpha) k t^{-\alpha}$$

$$\lambda_i = (1-\alpha) \lambda_c$$